Including virtual photons in strong interactions

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Abstract. In the perturbative field-theoretical models we investigate the inclusion of the electromagnetic interactions into the purely strong theory that describes hadronic processes. In particular, we study the convention for splitting electromagnetic and strong interactions and the ambiguity of such a splitting. The issue of the interpretation of the parameters of the low-energy effective field theory in the presence of electromagnetic interactions is addressed, as well as the scale and gauge dependence of the effective theory couplings. We hope, that the results of these studies are relevant for the electromagnetic sector of ChPT.

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1 Introduction

The systematic approach to take into account the electromagnetic corrections in the low-energy processes involving hadrons is based on the Chiral Perturbation Theory (ChPT) with virtual photons [1], which is the low-energy effective theory of the Standard Model in the hadron sector. Despite of been widely used in the applications, the procedure of the construction of the effective chiral Lagrangian with virtual photons from QCD is not free from conceptual difficulties. In particular, we mention the following points:

i) The effective Lagrangian with virtual photons operates with quantities like the quark masses m_q, the parameter B which is related with the quark condensate, the pion decay constant in the chiral limit F, etc. It is usually not specified, to which underlying theory – QCD+photons, or pure QCD – do these quantities refer. Obviously, if one interprets m_q to be the running quark masses in the full theory (including virtual photons), then the quark mass ratio m_u/m_d in ChPT Lagrangian turns out to be QCD-scale dependent. One encounters similar problem in relating B to the quark condensate, since $\langle 0|\bar uu|0\rangle$ and $\langle 0|\bar dd|0\rangle$ run differently for $e\neq 0$.

ii) According to the commonly used terminology, in the effective theory the combined contribution of the explicit photon loops and the electromagnetic effective couplings is called "electromagnetic corrections", and the rest is referred to as "strong piece". This implies, that exactly the latter survives when electromagnetic interactions are switched off. The problem is, however, how to rigorously define the theory with electromagnetic interactions switched off, even for the underlying QCD. To be precise, note that the initial theory with photons had four parameters: strong coupling constant g, fine structure constant α, and the quark masses m_u and m_d. The theory without photons has only three parameters: the strong coupling constant $\bar g$ which is obtained from g in the limit $\alpha\to 0$, and the quark masses $\bar m_u$, $\bar m_d$ to which m_u and m_d converge in the same limit. The question is, how to determine $\bar g$, $\bar m_u$ and $\bar m_d$ which define the theory in the limit $\alpha\to 0$? And, once this is done, how the above definition translates into the commonly used splitting of the low-energy effective theory?

iii) Although it has been pointed out [2,3], that some of the electromagnetic effective couplings do indeed contain scale- and gauge-dependence which is determined by the underlying QCD, a systematic study of this phenomenon is still lacking.

Although some of the issues mentioned here, have been already addressed and/or mentioned in the literature (see, e.g. [2,3]), a focused discussion of the conceptual problems in ChPT with virtual photons, to the best of our knowledge, does not exist so far. The aim of our investigations, which are very briefly surveyed in this work, is to set up notions for creation of such a coherent framework. We would like to stress in addition, that such investigations are an important ingredient in carrying out the consistent calculation of isospin-breaking corrections in the context of hadronic atom problem [4]. In order to demonstrate the general procedure, we do not consider QCD here – rather, we restrict ourselves to the simple perturbative models, where one may include electromagnetic interactions in a crystal clear manner. We believe, that the lessons one learns from these models, provide the necessary clue in a much more complicated case of QCD. A detailed discussion of the issues raised in this work, is given in the forthcoming publication [5].

2 Convention for the splitting

In this section, the following question is addressed: suppose, that one has the field theory which describes both electromagnetic and non-electromagnetic (referred hereafter as to the "strong") interactions. How does one systematically split strong and electromagnetic contributions in the quantities which are calculated within this theory? In order to answer this question, we find it useful to consider a simple perturbative model - the Yukawa model. The splitting convention which is explained in this model, is general and can be applied in the context of any other field-theoretical model.

The Yukawa model considered in this work, describes the doublet of "colored" fermions $\bar{\Psi} = (\bar{u}^i, \bar{d}^i)$, where the "color" index $i = 1, 2$. The fermions interact with the triplet of boson fields through the usual Yukawa coupling $\mathcal{L}_{\rm str} = g\bar{\Psi}\boldsymbol{\tau}\boldsymbol{\phi}\Psi$, which is characterized by a "strong" constant g. Further, the fermions whose charge matrix is given by $eQ \doteq e\,{\rm diag}(Q_u, Q_d)$, interact with the photon field in a standard manner, whereas the bosons described by the field $\boldsymbol{\phi}$, are neutral. For the renormalization, $\overline{\rm MS}$ scheme is used.

We demonstrate the idea of the splitting on the example of the physical mass of the fermion fields, given by the position of the pole in the propagator. Below, we restrict ourselves to the one-loop order. Denoting the masses by M_q where $q = u, d$, we find

$$M_q = m_q\bigl[1 + \frac{3}{16\pi^2}(3g_r^2 - 2e_r^2Q_q^2)\ln\frac{m_q}{\mu} + A_1 g_r^2 + A_2 Q_q^2 e_r^2\bigr] + O(g_r^4, e_r^2 g_r^2, e_r^4)\,, \qquad (1)$$

where $g_r,\ e_r$ denote the renormalized couplings, μ is the scale of the dimensional regularization, m_q stands for the running fermion mass. Further, A_1 is a known function of the boson-fermion mass ratio which is independent of the scale μ at this order (the explicit expression for this quantity is not needed), and $A_2 = (16\pi^2)^{-1}$.

The physical masses become scale independent, provided that the masses m_q run properly with the scale,

$$\mu\frac{dm_q}{d\mu} = \frac{3}{16\pi^2}(3g_r^2 - 2e_r^2Q_q^2)m_q + O(g_r^4, e_r^2 g_r^2, e_r^4)\,. \qquad (2)$$

The couplings g_r, e_r are scale independent at this order in the perturbative expansion. Once the running mass m_q is known at a some scale, the physical mass M_q is fixed in terms of the coupling constants g_r, e_r and of the running boson and fermion masses at this order in the perturbative expansion.

We now discuss the splitting of the physical masses into a strong and an electromagnetic part. This splitting should divide the mass into a piece that one would calculate in a theory with no electromagnetic interactions, and a part proportional to e_r^2: $M_q = \bar{M}_q + e_r^2 M_q^1 + O(e_r^4)$. Here and below, barred quantities refer to the theory at $e_r = 0$. The first term on the right hand side is

$$\bar{M}_q = \bar{m}_q\bigl[1 + \frac{9\bar{g}_r^2}{16\pi^2}\ln\frac{\bar{m}_q}{\mu} + A_1\bar{g}_r^2\bigr] + O(\bar{g}_r^4)\,. \qquad (3)$$

This part is scale independent by itself, provided that the mass $\bar{m}_q$ runs according to renormalization group equation (2) for $e_r = 0$. On the other hand, $\bar{g}_r$ is scale independent in this approximation. We see, that one has to fix a boundary condition in order to determine unambiguously $\bar{M}_q$. As a natural condition, we choose the running mass $\bar{m}_q$ to coincide with the running mass m_q in the full theory at a some scale: $m_q(\mu) = \bar{m}_q(\mu;\mu_1)$ at $\mu = \mu_1$ (here, we have explicitly indicated the μ_1-dependence of the barred quantities). With the use of the above matching condition, we express $m_q(\mu)$ through $\bar{m}_q(\mu;\mu_1)$, and insert the result into the expression for the mass M_q. Identifying g_r with $\bar{g}_r$ at this order, we find that

$$\bar{M}_q = \bar{m}_q(\mu;\mu_1)\bigl[1 + \frac{9\bar{g}_r^2}{16\pi^2}\ln\frac{\bar{m}_q}{\mu} + \bar{g}_r^2 A_1\bigr] + O(\bar{g}_r^4)\,,$$
$$M_q^1 = -\bar{m}_q(\mu;\mu_1)\bigl[\frac{6}{16\pi^2}\ln\frac{\bar{m}_q}{\mu_1} - A_2\bigr]Q_q^2 + O(\bar{g}_r^2)\,. \qquad (4)$$

This splitting has the desired properties: Each term on the right-hand side is scale-independent. However, as is explicitly seen in the contribution proportional to e_r^2, the splitting does depend on the *matching scale* μ_1. Indeed, one has

$$\mu_1\frac{d\bar{M}_q}{d\mu_1} = -\mu_1\frac{d[e_r^2M_q^1]}{d\mu_1} = -\frac{6e_r^2Q_q^2}{16\pi^2}\bar{M}_q\,. \qquad (5)$$

In other words, both terms in the splitting depend on the scale μ_1. This scale dependence is of order e_r^2 in the approximation considered. The sum M_q is of course independent of the matching scale.

A similar splitting may be considered for the running masses themselves. Indeed, expressing $m_q(\mu)$ through the running mass in the purely strong theory $\bar{m}_q(\mu;\mu_1)$ gives

$$m_q(\mu) = \bar{m}_q(\mu;\mu_1)\bigl[1 - \frac{6e_r^2Q_q^2}{16\pi^2}\ln\frac{\mu}{\mu_1}\bigr]\,. \qquad (6)$$

This result is the analogue of the relation (4) for the physical masses. It shows that the splitting of the running masses into a part that runs with the strong interaction alone, and a piece proportional to e_r^2, depends on the matching scale.

The dependence of the splitting on the scale μ_1 originates in the different running of the masses in the full theory and in the approximation when $e_r = 0$. This is illustrated in Fig. 1. The solid line refers to the running of the mass m_q in the full theory, whereas the dashed lines represent the running of $\bar{m}_q$. Because, for a fixed value of the scale μ, the running mass $\bar{m}_q$ depends on the matching scale chosen, the mass $\bar{M}_q$ does so as well.

One may wonder whether there is a way to split the pole mass in a unique manner. The reason why this is not the case is the following. In the Yukawa model considered here, the pole mass is proportional to m_q, which itself depends on the scale μ. In order to compare this mass with the corresponding quantity at $e_r = 0$, one has to compare two quantities that run differently, $\bar{m}_q$ and m_q. This running is itself a one-loop effect. There is therefore no possibility to avoid the ambiguity.

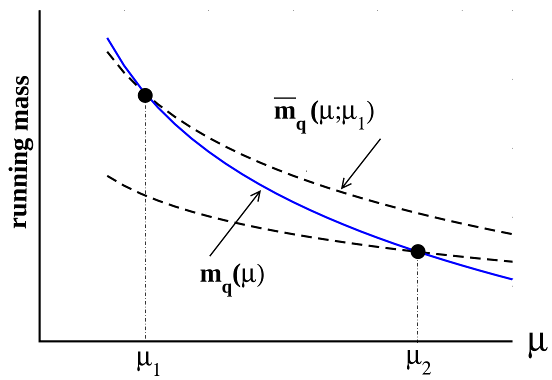

Fig. 1. The matching condition. The solid line represents the running of the mass m_q in the full theory according to (2), whereas the dashed lines display the running of $\bar{m}_q$.

Finally, we mention that the splitting of the parameters of the theory (masses and couplings) along the lines demonstrated above can be performed from a knowledge of the relevant beta-functions of the masses and of the coupling constants to any order in the perturbative expansion [5].

3 Splitting in the effective theory

On the example of the linear σ-model (LσM) with electromagnetic interactions we illustrate, how the prescription for splitting of electromagnetic and strong interactions does translate to the language of the low-energy effective theory. The calculations are done at one loop. In the limit of the large σ-mass, the model is equivalent to ChPT with the particular values of the couplings, expressed explicitly through the parameters of the initial model. This allows one to study the consequence of the splitting, performed in LσM, for the couplings of the effective theory. The dependence of these couplings on the renormalization scale in LσM, as well as on the gauge parameter, can be also studied. Below, we briefly list our conclusions obtained from these investigations. Detailed discussion is given in Ref. [5].

i) The splitting which is carried out in the underlying theory, is directly translated to the level of the low-energy effective Lagrangian of this theory. The μ_1-ambiguity of the parameters (masses and coupling constants) of the underlying *purely strong* theory which is matched to the theory with virtual photons at a scale $\mu = \mu_1$, is lumped in the couplings of the effective Lagrangian. These couplings have to be expressed in terms of the parameters of the purely strong theory and, possibly, some additional parameters that have to be introduced when the electromagnetic interactions are turned on. The advantage of doing so is, that the different parts of the Lagrangian then exactly describe the low-energy limit of the purely strong theory, and what is called the electromagnetic corrections. The price for this is just the above-mentioned μ_1-dependence of the effective couplings.

ii) When the electromagnetic interactions are turned on, some quantities like, e.g. the matrix elements of the vector current, start to be scale- and gauge-dependent. At the level of the effective theory, this dependence is systematically transformed into the scale- and gauge-dependence of the couplings of the Lagrangian.

iii) Most of the above conclusions can be straightforwardly applied to QCD without any change. The splitting of the quark masses, condensates, etc proceeds along the lines similar to those described in section 2. Further, the parameters of the low-energy effective Lagrangian of QCD in the strong sector refer to the pure QCD rather then to QCD+photons: e.g. the quark masses are the masses in pure QCD, etc. The price to pay is, that the couplings in ChPT depend on the matching scale μ_1. The (μ_1-dependent) results of calculations for any physical quantity, based only on the strong part of the effective Lagrangian, exactly reproduce the results that would be obtained for the same quantity in pure QCD matched to QCD+photons at $\mu = \mu_1$.

iv) It is important to note, that the μ_1-dependence puts natural limitations on the accuracy at which the couplings of the effective Lagrangian can be determined from the physical data which, of course, contains no μ_1 dependence. In principle, such a dependence should be observed if the couplings are theoretically derived from the underlying QCD e.g. via the lattice simulations.

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